**Semianalytical Solution for Thermo-Poro-Elastic Stresses in a Wellbore Cement Plug and Implications for Cement Properties that Minimize Risk of Failure**

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**Abstract**

Cementing materials used for plugging wellbores are subjected to evolving temperature, stress, and pore pressure conditions during their service lives. The induced pore pressure changes can be particularly problematic, especially in high temperature and high pressure (HTHP) environments and especially in low permeability materials. However, a design goal of most cement plugs is to achieve very low permeability, with the idea that lower permeability leads to better isolation. Here, with aid of a new semi-analytical solution for thermo-poro-elastic (TPE) stresses in a cylindrical cement plug that includes consideration of full coupling between hydraulic and thermal transport models (so-called “porothermoelastic-osmosis-filtration”, or “PTEOF” model), this work shows that lower permeability is not always better. Specifically, the solution shows that materials that are unable to drain excess pore pressure quickly enough compared to the rate at which these pressures build due to thermal changes, are more prone to generate regions of internal tensile effective stress and hence are more likely to be damaged. The specific parameter groups associated with this newly identified “permeability penalty” are obtained through a combination of dimensional analysis and pairwise bivariate analysis. These approaches give rise to two dimensionless groups of parameters that are mainly associated with propensity to generate TPE tensile effective stresses. The parametric space defined by these two groups is shown to have three distinct regions based on the probability of generating tensile effective stresses in a plug with a given set of material properties. By shifting the focus of material design from achieving the lower possible permeability to instead achieving the lowest permeability that will not incur increase likelihood of failure due to pore pressure buildup, this work provides a new design concept for wellbore cement. Furthermore, this work highlights for the first time the important role of specific heat of the cement in preventing pore pressure buildup, thereby showing a new way forward for cement design to increase this quantity.

Keywords: Cementing; Plug and Abandonment (P&A) ; High-temperature High pressure (HTHP); Fully coupled porothermoelastic-osmosis-filtration model; Permeability penalty; Phase-change cement;

1. Introduction

Properly designed and executed wellbore cementing and plugging operations are important for various Earth science-related geotechnical applications. These include radioactive waste disposal, deep-well plug and abandonment (P&A), drilling and completion in unconventional reservoirs, Enhanced Geothermal System (EGS), and carbon capture utilization and storage (CCUS) (Gruber et al., 2021; Hargis et al., 2021; Koťátková et al., 2017; Olson et al., 2015; Vrålstad et al., 2019). Despite the advancements in technological development of cementing materials over the last several decades, the quality of cementing is still often associated with some deficiencies, mainly due to the harsh environments where cement is placed (Ahmed et al., 2020; Allahvirdizadeh, 2020; Kiran et al., 2017). Taking wellbore P&A as an example, over the years and across companies, the upper range of the reservoirs’ pressure and temperature have been pushing up to 40000 psi and 600 Fahrenheit (DeBruijn et al., 2008; Khalifeh et al., 2020). However, the cement is originally designed for low-temperature and low-pressure conditions. Under harsh wellbore conditions, its stability over an extended period of time is unknown. To mitigate this problem, extensiveresearch has been focusing on reinforcing the cement by inclusion of various additives aiming to provide better mechanical and hydraulic properties, with the goal of maintaining the system integrity under the extreme conditions (Cai et al., 2022; Ge et al., 2018; Katende et al., 2020; Krakowiak et al., 2018; Massion et al., 2021; Massion et al., 2022; Qin et al., 2021; Samarakoon et al., 2022).

While much effort has been focused on development of materials and additives, the identification of what comprises “better” mechanical and hydraulic properties of cement are still unclear. This is especially true for high temperature and high pressure (HTHP) environments.

Wellbore cement can be classified as cementitious saturated porous material with permeability ranges from milli- to nano-Darcies (Banthia et al., 1989; Goto et al., 1981; Meng et al., 2021; Picandet et al., 2011). Such a material can be heavily influenced by thermo-hydraulic-mechanical (THM) coupling in the pore space, especially when it is experiencing large temperature and pressure variations. Within the permeability range mentioned above, a very large pore pressure could be induced by the THM coupling and the pore pressure would be progressively reestablished over the time, which will also lead to the changing of the effective stress and increase the possibility of shear failure, hydraulic fracturing, or even tensile failure (Ghabezloo et al., 2010). While THM coupling phenomena in porous media has been studied extensively, the wellbore-related applications have been mainly focusing on wellbore stability during drilling and fluid injection into borehole (Gao et al., 2017; Song et al., 2019; Tao et al., 2010; Zhou et al., 2009). The THM coupling effect in the cementing designs and operations has rarely been considered and fully investigated under the HTHP conditions. This knowledge gap means that conditions that will generate stresses that are able to damage the material are not well understood, and therefore it has potentially serious consequences.

To include these mutual interactions between thermal, hydraulic, and mechanical systems in the non-isothermal conditions, Biot (Biot, 1977) extended the traditional theory of poromechanics (Biot, 1941) to include the uncoupled thermal effects by incorporating the thermo-molecular diffusion and dynamic forces using a variational Lagrangian thermodynamics approach. Later on, the thermal diffusion process was coupled in solid and fluid deformation by Derski (1979), as well as others (Bear et al., 1981; Kurashige, 1989; Smith et al., 1993). The abovementioned porothermoelastic formulations include an assumption to neglect the non-linear term associated with connective heat transfer, which is thought to be most appropriate for low permeability materials (Chen et al., 2005; Delaney, 1982; Gomar et al., 2014; Wang et al., 2003). Within the framework of linear porothermoelasticity, substantial studies have been performed for coupled THM behavior of isotropic porous media, but most of the works are assuming the fluid flux and heat flux are dominated by the pore pressure gradient and thermal gradient, respectively (Ghassemi et al., 2002; Ghassemi et al., 2009; Valov et al., 2022). That is to say, the thermo-osmosis denoted by (fluid flux generated by thermal gradient) and mechano-caloric effects denoted by (heat flux generated by pore pressure gradient) are neglected in the transport equations

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Here is denoting the fluid flux and is the heat flux, while *p* is the pore pressure and *T* is the temperature field. Noted that the mechano-caloric coefficient is also known thermal filtration coefficient (Cheng, 2016).

Although thermo-osmosis and mechano-caloric effects are often neglected (if for no other reason, this assumption greatly simplifies solution methods), for porous material with low permeability, these two effects can play important roles (Gonçalvès et al., 2010; Roshan et al., 2015; Trémosa et al., 2010). For example, Carnahan (1983) has shown that the thermo-osmosis flow through kaolinite can be two orders of magnitude higher than Darcy’s flow (that is, the fluid flux driven by the pressure gradient term in Eq. (1)) near a nuclear waste repository. Thus, when designing the cementing under the HTHP conditions, both the thermo-osmosis and mechano-caloric effects should be taken into consideration and should not be dismissed at the outset of the solution. To the best of our knowledge, under the HTHP conditions, their influences on the cement integrity are still unclear.

By introducing the so-called fully-coupled porothermoelastic model, which incorporates both of the thermo-osmosis and the mechano-caloric (thermal filtration) effects, dubbed here as “porothermoelastic-osmosis-filtration” (PTEOF), the present work uses a cylindrical geometry and boundary conditions inspired by cement plugs for P&A as an example to highlight the cementing challenges that are associated with HTHP conditions. The motivations of creating the PTEOF model is to have a comprehensive understanding of the cement’s behaviors under the HTHP and to build up a general framework and solutions for future cementing studies and analysis. Specifically, we derive a semi-analytical solution (analytical up to a numerical inversion of a Laplace transport) that draws on the method of Sarout et al. (2011) and therefore leverages the mathematical similarity between PTEOF and linear chemo-poroelasticity. After presenting the governing equations, solution method, and examples of the behaviors predicted by the model, the key parameters associated with preventing tensile effective stresses from developing in a cement plug are identified. These are identified through a combination of dimensional analysis and pairwise bivariate analysis, leading to dimensionless groups that define a parametric space with regions that are “safe” and regions that are associated with material parameter combinations more likely to sustain damage. The work concludes with a discussion of implications for design of cement materials that do not just pursue the lowest possible permeability, but rather pursue a combination of material properties that will provide the necessary isolation without incurring elevated risk of damage from PTEOF phenomena under HTHP conditions.

2. Governing Equations

Following the sign convention in Detournay et al. (1988), positive stress is considered to be tensile within the present work. Then the tensorial strain-stress constitutive relation that expatiates upon the coupled thermo-hydro-mechanical behaviors of fluid saturated porous medium could be rewritten as

, (3)

where and is stress tensor components and strain tensor components, respectively;is pore pressure change from virgin pore pressure and is temperature change from the reference temperature ; is shear modulus and  is Poisson’s ratio;  is the Biot effective stress coefficient and  is the thermoelastic effective stress coefficient.

Based on quasi-static equilibrium

, (4)

and the strain-displacement relations

 , (5)

and by substituting both transport laws (Eq. (1) and Eq. (2)) into the conservation laws

, (6)

, (7)

the fully coupled diffusion equations

, (8)

, (9)

can be obtained. These two diffusion equations indicate that both fluid flux and heat flux are not only dominated by Darcy’s law and Fourier’s law, but that they are also influenced by the thermal osmosis effect and thermal filtration effect. This is also where porothermoelastic-osmosis-filtration (PTEOF) so called fully coupled model roots in. One of the motivations to build up this fully coupled model is to facilitate further studies and analysis. For example, under some circumstances where thermal osmosis or thermal filtration are not considered important, the corresponding coefficient can always be taken as zero to simplify the model.

Based on the fundamental work of Cheng (2016) and Wang (2017), within the framework of linear thermoporoelasticity, the coupled volumetric response relations are

, where , (10)

where one of these three equations can be obtained from contraction on Eq. (3). In Eq. (10),  and  is volumetric strain tensor and total stress tensor, respectively;  is the variation of fluid content per unit volume;  is entropy density. The material constants include the drained bulk modulus tensor, Biot effective stress coefficient, Skempton pore pressure coefficient, coefficient of volumetric thermal expansion of porous media frame  , coefficient of volumetric thermal expansion of variations in fluid content in the solid-fluid system  (Cheng, 2016), and  represents the specific heat of the porous medium at the reference temperature.

From the above constitutive equations (Eq. (10)), it can be seen that the deformation of the solid frame is caused by changes in stress, pore pressure, and/or temperature. The fluid phase in the porous medium is not only deforming with the solid frame, but at the same time, driven by pore pressure gradient and thermal forces, causing the pore fluid entering or leaving the solid frame of unit volume. Last, the stress and temperature change will cause the change of the entropy of the porous system based on the generalized-energy relation. The entropy density is therefore becoming a function of volumetric strain of the solid frame, fluid content, and the change of temperature. Thus, the constitutive equations relate and couple volumetric strain, fluid content, and energy variables  with total stress, pore stress, and temperature variables with the material constants.

3. PTEOF solution for the primary cementing in P&A

3.1 Problem descriptions and boundary conditions

The present work will showcase the PTEOF model by using the primary cementing in P&A cases where the length of the primary plug is usually 50 to 100 times larger than its diameter (Eshraghi, 2013); it is appropriate to apply the generalized plane-strain assumption where the pore pressure and thermal diffusions only appear in the isotropic plane that is perpendicular to the length axis of the plug which is fully saturated. In line with the loading decomposition scheme proposed by Detournay et al. (1988) in the poroelasticity, the PTEOF model could be decomposed into three sub-loading cases to simplify the analysis, i.e. pore pressure loading (), temperature loading (), and isotropic far-field stress loading (), where the superscript i is denoting by the stress field that induced by the loading mode j which is the subscript. Thus, the boundary conditions at the outside surface of the primary cement plug for each of the loading modes can be written as follows (the subscript f denotes formation, subscript c denotes cement)

* Mode 1: ,, and ;
* Mode 2: ,, and ;
* Mode 3: ,, and .

Since the PTEOF model is linear, the principle of superposition will be used as final step to obtain the final solution.

3.2 Solution to the fully coupled diffusion equations

The solution method starts with obtaining a general solution of the fully coupled diffusion equations (Eq. (8) and Eq. (9)). Note that the plane-strain assumption will lead to the conclusion that fluid and thermal transport will be only directed along the radial direction in the cylindrical coordinates system. Then, the first step is to eliminate  and  in the diffusion equations by substituting the constitute equation (Eq. (10)), this results in a new form of coupled diffusion equations

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Next, by combining Eq. (3), Eq. (4) and Eq. (5), and taking the body force to zero, an extended form of the classical Navier equation about be obtained as

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Drawing on the important contributions of Sarout et al. (2011) which modify the theory of linear chemoporoelasticity into a convenient form, whereby the interpretations of the phenomenological parameters can be clarified, we can then use the irrotational field assumptions assumptions to simplify the process of solving the Navier equations. When displacement field is irrotational, i.e. is the gradient of a scaler, one can integrate Eq. (13) to obtain

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where  and .

Thedoes not depend on the spatial coordinates and it is a spatially uniform function that often zero for infinite or semi-infinite domain (Detournay et al., 1993). But in the case of primary cementing in P&A, thewill not be taken as zero here. Next, by substituting Eq. (14) into Eq. (11) and Eq. (12), the  can be eliminated and it can have the coupled diffusion equation

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where:

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The diffusion equations in Eq. (15) are coupled in terms of T and p, which can be transformed into uncoupled equations in terms of  and by using the Eigen decomposition approach developed by Sarout et al. (2011). This methodology starts by obtaining two eigenvalues  and  of matrix. Then, the Eigen-decomposition theorem (Weisstein, 2002) allows us to define a transition matrix, which is composed of eigenvalues and eigenvectors of , and temperature and pore pressure are

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where

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This transition leads directly to an uncoupled system of diffusion equations

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where . Then, by applying the Laplace transform to the Eigen function

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where , it becomes an ordinary differential equation in terms of variable , where  is function of the coordinators in cylindrical system ρ, and the Laplace parameter s and the ’s eigenvalue .

Therefore, the original coupled diffusion equation in Eq. (15) has been lead to the zeroth-order modified Bessel equation, with the general solution

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where  and  are unknowns that will be determined based on boundary conditions. And are the zeroth-order modified Bessel functions of the first and second kind, respectively.

The symmetry conditions of the primary cementing in P&A will ensure that equals to zero, thus the pore pressure and temperature profile in the Laplace domain are

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Within the framework of plane-strain conditions and irrotational field conditions, the displacement, , can be obtained by integrating the Eq. (14). At the same time, the strain-displacement relationship are

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where  and .

Up to now, there are three unknowns  in total, i=1,2,3 in the system. The first two unknowns  and come from the coefficient of the zeroth-order modified Bessel functions of the first kind and the third unknown is coming from the spatially uniform function from Eq. (14) in the Laplace domain. In the process of deriving the above expressions for the pore pressure, temperature, and radial total stress (all in the Laplace domain), we can create three equations for these three unknowns

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where

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So when given a value of s, the  can be readily computed, thus the solution can be numerically inverted to the time domain using Stehfest’s method (Stehfest, 1970), which has been proved to be efficient in poroelastic problems, and its details are shown in Appendix B.

4. Parametric Analysis and Discussions

The notations that are used for the modeling are summarized in Table 1 in Appendix A. The temperature difference between the formation and cement is assigned as; the virgin pore pressure of the formation is setup as Pa and far-field isotropic stress is setup as Pa (Snee et al., 2018; Xu et al., 2015; Zoback et al., 2003). In this section, we will firstly discuss the pore pressure profile and temperature profile that are induced by three different loads, respectively. Lastly, the superposition results of the effective stress will be summarized.



4.1 Pore pressure responses and thermal osmosis effect

The change of pore pressure in response to different loadings is a key component in poromechanical analysis. It is therefore of interest to firstly plot the evolution of pore pressure under three different loading modes, given the boundary conditions that are defined in the last section, while the influence of the factors of interest on the pore pressure will be analyzed and discussed later.

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Figure 1a. Pore pressure change due to mode 1 loading, where the curves grade from gray to black as time increases

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Figure 1b. Pore pressure change due to mode 2 loading, where the curves grade from gray to black as time increases

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Figure 1c. Pore pressure change due to mode 3 loading, where the curves grade from gray to black as time increases

Fig.1a and 1b illustrate the evolution of the pore pressure profile along the radius in response to mode 1 loading (pore pressure loading) and mode 2 loading (temperature loading), respectively. Under the mode 1 loading, it is similar to the classical diffusion process; the pore pressure near the surface (r=1) is instantly raised to the level of pore pressure loading, and then the pore pressure is gradually diffused towards the center (r=0). However, in contrast to gradual and smooth diffusion process, the pore pressure induced by mode 2 firstly peaks near the surface area (r=1). At the surface, the pore pressure returns to zero due to the boundary conditions setup. The pore pressure, however, decreases toward the inner core, because of the slow diffusion progress due to low hydraulic conductivity, hence a peak profile is developed. As time progresses, the pore pressure peak declines due to fluid diffusion, and at the same time, the inner core becomes heated as well, and so the peak is moving towards the center. At even later times, when the entire specimen is heated but the pore pressure is not yet dissipated due to the small fluid diffusivity, as time further increases, the entire pore pressure dissipates and returns to zero everywhere.

For the pore pressure that is induced by mode 3 (isotropic far-field stress), as shown in Fig.1c, it firstly arrives the highest value right after the loading (again, pore pressure at the surface reduces to zero due to the boundary conditions setup), but it gradually declines to zero due to the dissipation process as the time processes. It should be noted that the magnitude of the pore pressure induced by mode 3 is much smaller compared to the pore pressure induced by mode 1 and mode 2. Even at the highest level of pore pressure which is right after the loading is applied, the ratio of mode 3 induced pore pressure over mode 1 and mode 2 induced pore pressure is around 0.01%-0.02%. This is also in line with the findings from the classical pure poroelastic model that Detournay et al. (1988) concluded.

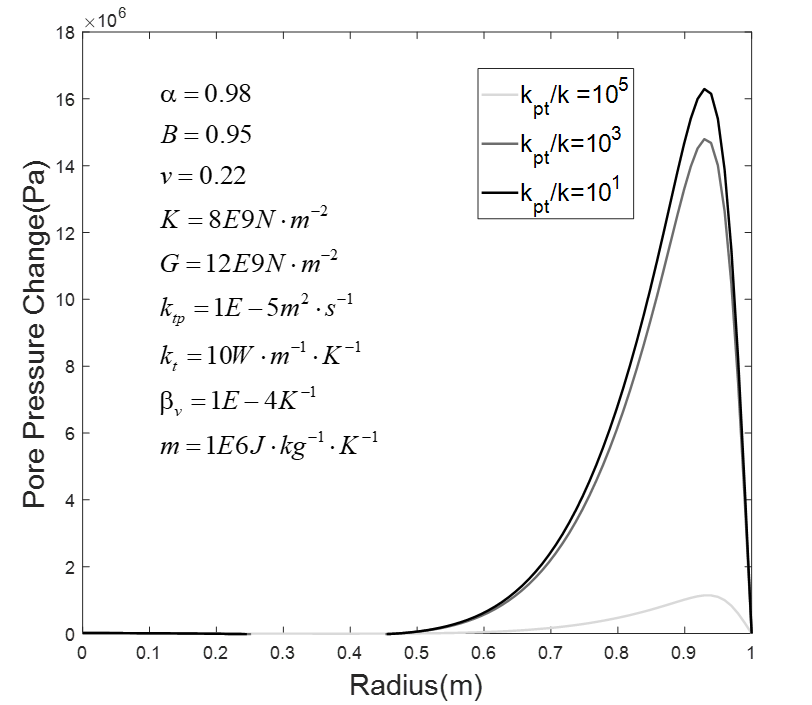


Figure 2. Various ratio of (thermo-osmosis’s influence) on the induced pore pressure (days)

Since the peak pore pressure usually occurs at an early time, we will use the smaller time at days after loading to illustrate the thermal osmosis effect on the induced pore pressure profile. Three different ratios (1, 103, 105) of  are selected to show the different developments of the temperature induced pore pressure (Figure 2). Recall that the definition of the thermal-osmosis coefficient is understood as the passage of a fluid that is driven by a temperature gradient, but against the hydrostatic pressure (Denbigh, 1949). Taking the early time as an example, all three cases show a peak pore pressure near the surface region; however, the magnitude of the peak induced pore pressure is reduced when the thermos-osmotic coefficient is significantly larger than the hydraulic conductivity (i.e., =105).

4.2 Temperature response and thermal filtration effect

The temperature profiles under the mode 2 loading (temperature loading) are shown in Figure 3. As the time proceeds, temperature is increasing monotonically from surface towards the inner core of the plug, and finally arrives at equilibrium, which is the equilibrium of the boundary conditions at the later time. Three different ratios of thermal conductivity over the thermal filtration coefficients are selected to show the impact of the thermal filtration effect on the temperature profile, that is changed by the pore pressure gradient. As shown in Figure 4, among the three selected ratios, the maximum temperature difference induced by the pore pressure gradient is within 1 degree Kelvin. Furthermore, when the thermal conductivity is significantly larger than the thermal filtration coefficient (=105), the induced temperature differences are negligible under the current model settings and inputs.

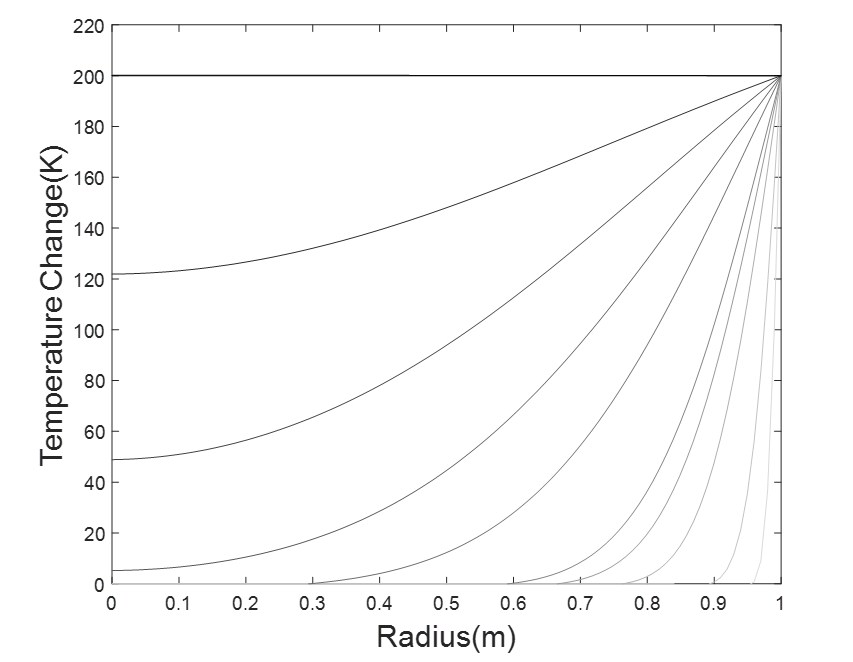


Figure 3. Temperature profile development under the loading mode 2, where the curves grade from gray to black as time increases

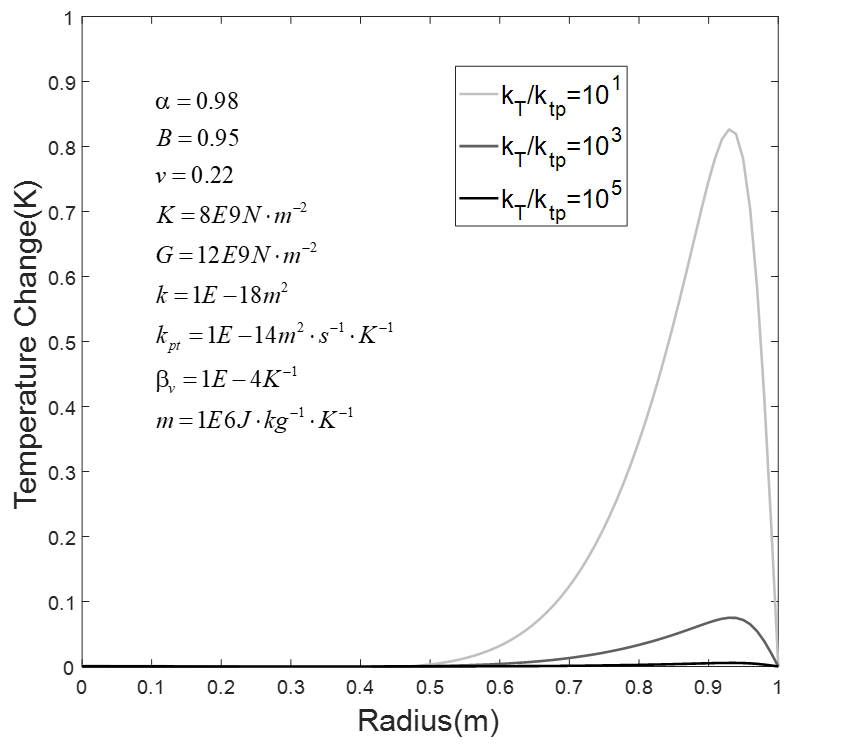


Figure 4. Various ratio of (thermal filtration’s influence) on the temperature profile change

4.3 Effective Stress

Many pioneering researches have shown that the effective stress is a key factor in controlling the behaviors of porous medias in the geotechnical and earth applications (Khalili et al., 2004; Skempton, 1984). Specifically, increasing pore pressure will reduce the effective stress, which can increase the possibility of many types of failures. Thus, appropriate estimation and calculation of total effective stress will greatly enhance the understanding of the behaviors of the porous system. As shown in figure 5, where the total effective radial stress (after superposition from three loading modes) is plotted (again, tensile is positive). After the cement is placed under the boundary conditions mentioned previously at the early stage, the induced pore pressure will reduce the radial effective stress near the surface region, but the entire sample will still be under the compressive condition. However, because the diffusion process is very slow inside the sample, but the pore pressure is still accumulating due to the HTHP conditions, the effective stress will be further reduced, and a tensile region will be created inside the core. At a later time when induced pore pressure is fully dissipated, the radial effective stress will return to compressive again. This demonstration of the whole diffusion process of the PTEOF model with given inputs, indicates that cementing under the HTHP will definitely induce a high value of pore pressure, which will lower the effective stress and even generate a tensile region as the results of pore pressure slowly diffuse inward. These increases the possibility of inducing different kinds of failures in the cement and jeopardizes the integrity of the cementing system, which will cause more severe consequences.



Figure 5. Total effective stress of cement under the conditions of HTHP, where the curves grade from gray to black as time increases (boundary conditions and input values of each parameter are stated at the beginning of this section)

4.4 Full parametric studies

It is worth mentioning that the system behaviors discussed in the last section depend on the different values of input parameters and the mutual interaction and group effect among them. If some of these parameters take on different values, the results can be much different. Thus, these phenomena will be investigated and discussed in this section. The analysis starts with assigning values (within a certain range that is summarized in Table 2 in Appendix A) to each parameter by the Monte Carlo method, which is a class of techniques for randomly sampling (Shapiro, 2003). Then, once these variables have been given values, they will be substituted into the constitutive equations and loading decomposition procedures discussed previously to obtain the effective radial stress, which will be categorized into tensile and compressive groups. The details of the above-mentioned procedures are summarized in the flowchart in Figure 6.

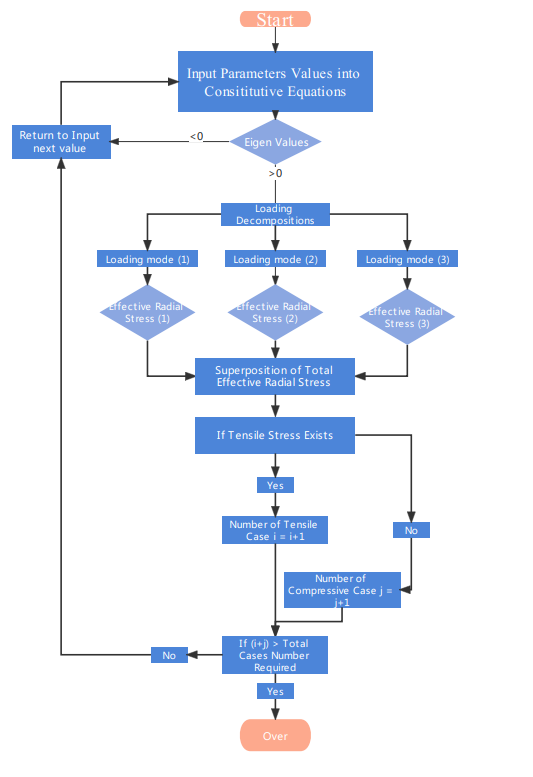


Figure 6. The Algorithm that is used to construct the fully parametric studies

4.4.1 Pairwise Bivariate Analysis for Each Variable

Following the procedures the flowchart described in Figure 6, a total of 3000 cases are calculated and then categorized into two groups: tensile (1727 cases) and compressive (1213 cases). Next, the relationship between each variable is firstly shown by the pairwise bivariate distributions that are shown in Appendix C. The non-diagonal elements are scatter plots which display the correlation between two variables and give a valuable insight on the distribution features of these variables. The lower and upper triangles formed by the diagonal are symmetrical. The diagonal elements are univariate distribution plots which are drawn to show the probability density function of each variable. It can be found that in the events of a tensile case, the specific heat and permeability are more concentrated at their lower range, and the thermal conductivity and the thermal expansion coefficient differences are more concentrated at their higher range, whereas the rest of the parameters are almost evenly distributed along their whole range. This indicates a general trend that, within the framework of PTEOF, at higher values of  and lower values of  and  will increase the probability of generating tensile radial effective stress increases.

A very interesting point can be found and discussed here. One current cement designing principles are that the lower hydraulic conductivity, the better the system behavior. This cement designing philosophy is effective under most working conditions (i.e., ambient temperature and pressure) that cement is originally built for. However, when the cement is placing under the HTHP conditions, and poromechanics are taken into considerations, our results show that the lower permeability will actually increase the probability of the tensile case, which is considered detrimental to the integrity of the cementing system. This is mainly due to the fact that under the framework of porous media, the lower permeability will greatly slow down the pore fluid diffusion process when the pore pressure is rapidly built up within the cement due to the HTHP boundary conditions. The accumulated pore fluid within the clogged porous system will build up excess pore pressure and the effective stress will thus be reduced or even extended into the tensile region which will eventually jeopardize the whole system. That is to say, blindly pursing low permeability regardless of the intrinsic porous properties of the cement itself will instead victimize the cement functions that are providing mechanical support and zonal isolations. This phenomenon is firstly named here as permeability penalty. To avoid permeability penalty for cementing design under HTHP, perhaps the most favorable solution is to keep the permeability at certain ranges which can achieve the sealing function, but at the same time, it allow the diffusion of pore pressure that is built up by the HTHP conditions and therefore not cause unnecessary damage induced by the excess pore pressure. This permeability-forgiveness design is actually very popular in pavement design of permeable porous systems (PPS) (Scholz et al., 2007) where one or two special drainage layers with relative higher permeability are designed in a more effective and efficient way that reduces the runoff rates in the storm while providing a hard surface for the traffic flow. The PPS design has been successfully turned into wide variety of residential, commercial, and industrial applications in the last two decades (Drake et al., 2013).

It also worth mentioning that the specific heat capacity is rarely got any attention in previous research of HTHP cementing design. However, based on our results, it does play a very important role, similar to the role of thermal conductivity, as a large specific heat capacity will slow down the heat transfer process and the thermal induced pore pressure build up progress will be delayed, which gives the pore fluid more time to dissipate. This provides a totally new perspective for creating a phase-change cement with high volume specific heat capacity but low thermal conductivity that is specially designed for HTHP conditions, which will have very promising and wide applications.

4.4.2 New Proposed Dimensionless Parameters

Although the pairwise bivariate distributions shown in Appendix C give a full picture of how each parameter will individually influence the system behaviors, the mutual interactions of different properties and their group effect cannot be analyzed from it. To solve this problem, scaling analysis is one the classical methods that is utilized. It is usually used to reduce a complex physical problem to a simpler version prior to obtaining a quantitative answer while also grasping the effects of various physical phenomena at the same time . We indeed can follow the typical procedures used to conduct the scaling analysis to investigate the mutual interactions and group effect of the PTOEF model. However, the new dimensionless numbers that result from the scaling process are often in very complicated expressions and lack a direct meaning at the physical level, and therefore can be very ambiguous in terms of many engineering applications. Thus, inspired by the spirit of the methodology of scaling analysis, we have adapted the results from the scaling analysis to manually compose two new composite dimensionless parameters with clearer physical meaning, and then construct a new plot by using these two parameters. It helps us to distinguish different system behaviors that can account for the mutual interactions and group effects among these different parameters. Among these different parameters, therefore making it easier for many cementing practitioners to pick them up and apply them to the real engineering problem. Thus, two new parameters are hereby proposed as follows:

The first dimensionless number,, consists of the Biot coefficient, Skempton coefficient, thermal expansion coefficient of fluid and solid, as well as thermal diffusivity and fluid diffusivity, where the last two terms are obtained from the diagonal elements of the coefficient matrix in Eq. 15. This new proposed mainly contains the hydro-thermal properties of the porous media. It is thus named the Hydro-Thermal Non-Dimensional parameter (HTND). The second proposed parameter,, consists of three mechanical related components: Shear Modulus, Bulk Modulus, and Poisson’s ratio. Thus, it is named the Mechanical Non-Dimensional parameter (MND).

Given the new proposed parameters, a new plot can by generated to show the tensile and compressive spatial distribution cases based on( HTND) and(MND). In Figure 8 (a), the same 3000 cases are replotted where the red dot represents the tensile case and the green dot represents the compressive case. It can be clearly seen that the spatial distributions of the tensile and compressive cases in the- plot heavily depend on the whereas both tensile and compressive cases are almost evenly distributed along theaxis. This indicates that the influence of on the system dominates when compared with . Futhermore, in Figure 8 (b) where is equally divided into 300 intervals, the probability of generating the tensile cases in each of these intervals is calculated. It is shown that in the area where theare smaller than 0.07 is fully occupied by compressive cases and single tensile case exists in this region. It is thus called the safe zone, which means no tensile failure would occur within this range under the framework of the PTEOF model. When  increases from 0.07 to 1, the probablity of generating tensile cases gradually increases up to 100%. Both tensile and compressive cases could happen in this region, but with a higher value of , the possibility of generating a tensile case will be higher. This area is thus called the transition zone. When is greater than 1, no compressive case is exists within this range and it is fully occupied by tensile cases, which means the probality of generting a tensile cases is 100%. It is thus named the damage zone which means when designing the cement under the HTHP conditions, if the resulting is landing greater 1, tensile damage will certainly occur in the cement.

Drawing on the conclusions above, it can be easily seen that the Bulk Modulus, Shear Modulus and Poisson’s ratio, which are considered as three of the most important mechanical properties, have very limited contribution to the system behavior when in terms of  is presenting. This actually challenges the current HTHP cementing design principles which always prioritize making the cement stronger and more ductile, hoping that the cementing system will thus have more resilience and less possibility of failure. However, based on our results, this actually may not help to maintain the cement integrity under the HTHP conditions. That is to say, when designing the cement under the HTHP conditions, rather than give the highest priority to these mechanical properties, more attention should be put into the components that keep the value of below 0.07, as to avoid any tensile damage.

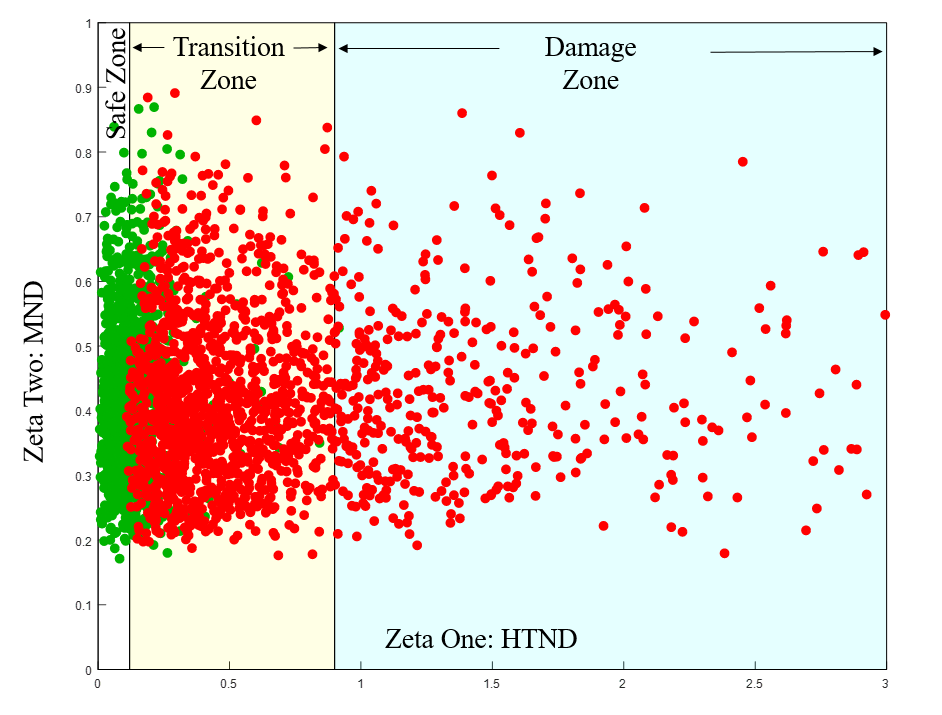


Figure 8 (a) (HTND) and(MND) plot with tensile cases (red color) and compressive cases (green color).

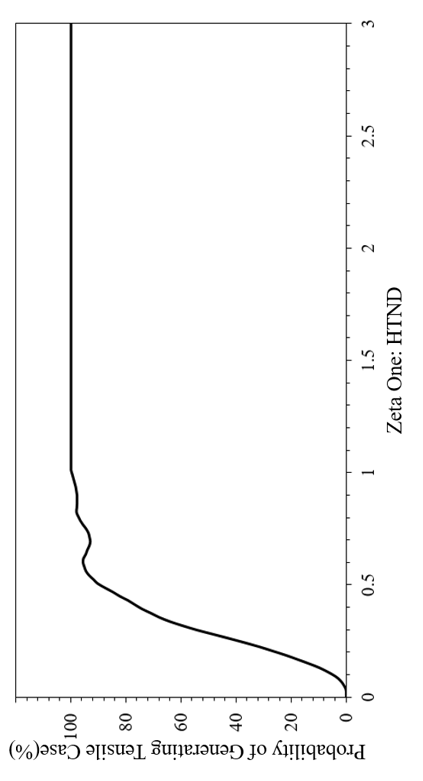


Figure 8 (b) The probability plot of generating tensile cases on different  (HTND) values

5. Conclusions

Using the primary cementing in P&A as an example, the fully coupled porothermoelastic-osmosis-filtration (PTEOF) model is presented here. It has been demonstrated that under HTHP conditions, the induced pore pressure in low permeability material such as cement can be so large that it greatly reduces the effective stress, and can even extend the cement into the tensile region. This reduced effective stress phenomenon will, in general, increase the probability of cement failure that may occur in different types. By the pairwise bivariate analysis, the properties that are important for the HTHP cementing are parsed out. Based on these results, two new concepts of permeability forgiveness cement and phase-change cement are proposed here for the future of HTHP cementing design. Furthermore, by adapting the scaling analysis approach, three zones (safe, transition, and damage) with clear boundary values are shown in the (HTDN) and (MDT) plot, to give the cementing researchers and practitioners a direct and meaningful guidance about how much tensile failure risk the cement is facing when it is placed under the HTHP conditions.

More importantly, the implications of our results challenge the prevailing HTHP cementing principles and provide a new perspective of design and guidance by addressing the porous intrinsic nature of cement. Current cementing improvement practices, such as adding different additives into cement to enhance its mechanical properties, will not change the fact that the new additive mix is still a porous media. While the new additive can change the pore space into various sizes as small as the nano-scale, without considering the underlying physical principles governing the mechanical behaviors under the framework of porous media, it sometimes will jeopardize the whole system instead. Our results indicate that when cementing under HTHP conditions, the mechanical properties, such as shear modulus and Poisson’s ratio, may not be the most important parameters to maintain the integrity of cement, and the low permeability will actually cause the tensile stress to damage the system. Thus, we are proposing a new cementing design principle of permeability forgiveness, which allows for some tolerance of permeability in order to avoid the permeability penalty.

Another important implication from our results is that instead of focusing on enhancing the mechanical properties, thermal properties, such as thermal conductivity, specific heat capacity, thermal expansion coefficient ratio, and the thermo-osmosis coefficient also deserve more attention. Our analysis shows that the magnitude of thermal induced pore pressure results from the competing process of thermal diffusion and fluid diffusion. Thus, slowing down the thermal transfer process becomes a key step in preventing the thermal load induced pore pressure damage. Certainly, reducing the thermal conductivity is the most direct method to achieve this goal. However, our results show that increasing the specific heat capacity is another efficient way to slow down the pore pressure build up progress. More importantly, it provides a totally new perspective of creating a phase-change cement with high volume specific heat capacity but low thermal conductivity that is specially designed for HTHP conditions, which has very promising and wide applications.

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Appendix A: Table 1. Notations

|  |  |  |
| --- | --- | --- |
| Symbol | Definition | Unit |
|  | Biot effective stress coefficient | --- |
|  | Drained thermoelastic effective stress coefficient | N⋅m-2⋅K-1 |
|  | Skempton pore pressure coefficient | --- |
|  | Coefficient of volumetric thermal expansion of solid | K-1 |
|  | Coefficient of volumetric thermal expansion of porosity | K-1 |
|  | Coefficient of volumetric thermal expansion of fluid | K-1 |
|  | Coefficient of volumetric thermal expansion for variation in fluid content | K-1 |
|  | Drained coefficient of volumetric thermal expansion of porous medium frame | K-1 |
|  | Specific heat of the porous medium at reference temperature | J⋅kg−1⋅K−1 |
|  | Fluid flux | m⋅s−1 |
|  | Heat flux | J⋅m−2⋅s−1 |
|  | Permeability | m2 |
|  | Thermal conductivity | W⋅K−1⋅m−1 |
|  | Mechano-caloric coefficient (Thermal filtration) | m2⋅s−1 |
|  | Thermo-osmosis coefficient | m2⋅s−1⋅K−1 |
|  | Poisson’s ratio | --- |
|  | Drained bulk modulus | N⋅m-2 |
|  | Shear modulus | N⋅m-2 |
|  | Thermal Diffusivity | m2/s |
|  | Fluid Diffusivity | m2/s |

Table 2. Lower boundary and upper boundary for Monte Carlo Sampling (the unit of each parameter is the same as the unit defined in Table 1)

|  |  |  |
| --- | --- | --- |
|  | Lower Boundary | Upper Boundary |
|  | 0.55 | 0.95 |
|  | 0.58 | 0.98 |
|  | 1E-5 | 1E-4 |
|  | 8E9 | 16E9 |
|  | 12E9 | 24E9 |
|  | 0.22 | 0.32 |
|  | 1E-17 | 1E-16 |
|  | 1E1 | 1E2 |
|  | 1E-5 | 1E-4 |
|  | 1E-14 | 1E-13 |
|  | 1E6 | 1E7 |

Appendix B: Details of Stehfest’s method

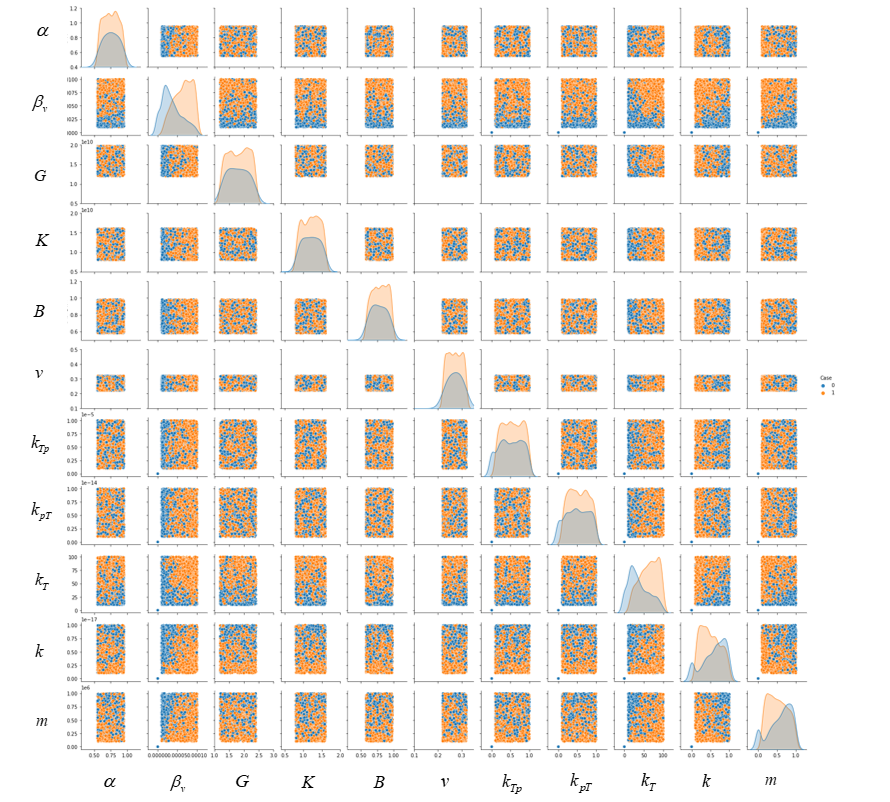
For a given function p with Laplace transform, Stehfest’s method can be expressed as, taking pore pressure for example:



Where the coefficient are given by



Appendix C:



Pairwise bivariate distributions for eleven variables (each orange color dot represents a tensile case and each blue color dot represents a compressive case)

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